Week 5: Functions

Data Analysis for Psychology in R 1

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Course Overview

Exploratory Data Analysis	Research design and data
	Describing categorical data
	Describing continuous data
	Describing relationships
	Functions
Probability	Probability theory
	Probability rules
	Random variables (discrete)
	Random variables (continuous)
	Sampling

Foundations of inference	Confidence intervals
	Hypothesis testing (p-values)
	Hypothesis testing (critical values)
	Hypothesis testing and confidence intervals
	Errors, power, effect size, assumptions
Common hypothesis tests	One sample t-test
	Independent samples t-test
	Paired samples t-test
	Chi-square tests
	Correlation

Weeks Learning Objectives

1. Understand the basic principles of functions.
2. Understand concept of data transformations.
3. Understand the calculation of z-scores.

Topics for today

- What is a function?
- Linear and non-linear functions
- How do we use functions in statistics?
- An example of z-scores

What is a function?

- A function takes an **input**, **does something**, and provides an **output.**
- **Input**

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- **Doing something**

$$f(x) = x - 2$$

- **An output**

$$y = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Functions and relations

- It is important to think of the function as showing the *relationship* between input and output.
- We can link this to the idea of relationships from week 4.
- The function links an input (predictors, x), to an output (outcome, y)
- So we can write

$$y = f(x) = x - 2$$

Visualising Functions

- An important tool in understanding functions is to plot them.
- So let's look at the following:

$$y = f(x) = 10 + 2x$$

Visualising Functions

- Our input x is a vector of numbers:

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

Visualising Simple Functions

x	y
1	12
2	14
3	16
4	18
5	20
6	22
7	24
8	26

```
func_x <- tibble(
  x = c(1,2,3,4,5,6,7,8),
  y = 10 + (2*x)
)
```

- `tibble` is used to create a data set
- `x` is our original data entered as a vector of numbers using `c()`
- `y` is the output of the function `f(x) = 10+(2*x)`

Visualising Simple Functions

x	y
1	12
2	14
3	16
4	18
5	20
6	22
7	24
8	26

$$y = f(x) = 10 + 2x$$

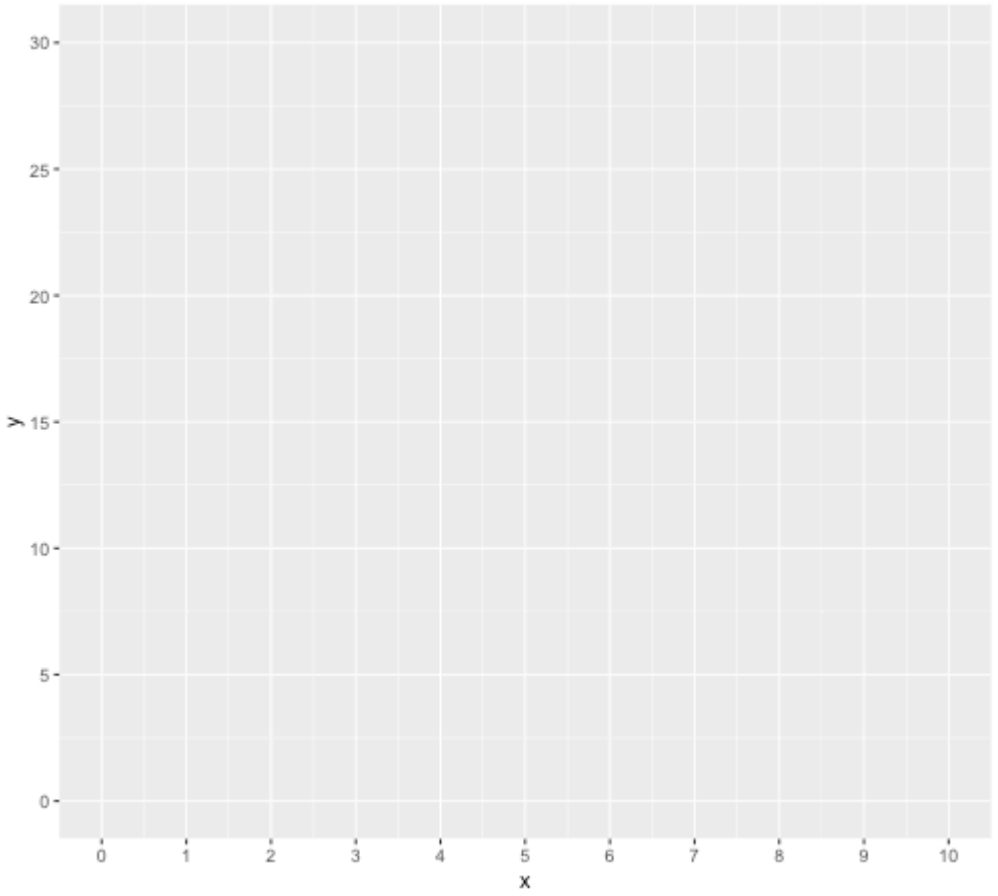
- Example row 1:

$$10 + (2 * 1) = 12$$

- Example row 5:

$$10 + (2 * 5) = 20$$

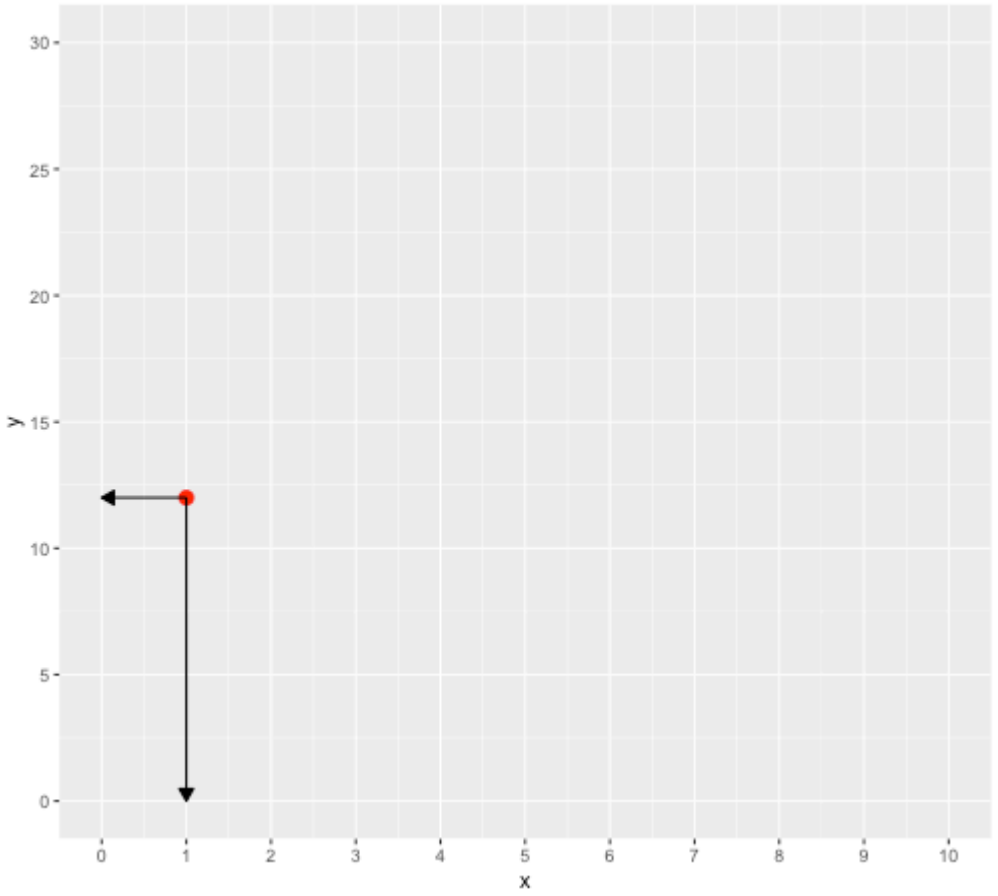
Visualising Functions



Our Data

x	y
1	12
2	14
3	16
4	18
5	20
6	22
7	24
8	26

Visualising Functions



Our Data

x	y
1	12
2	14
3	16
4	18
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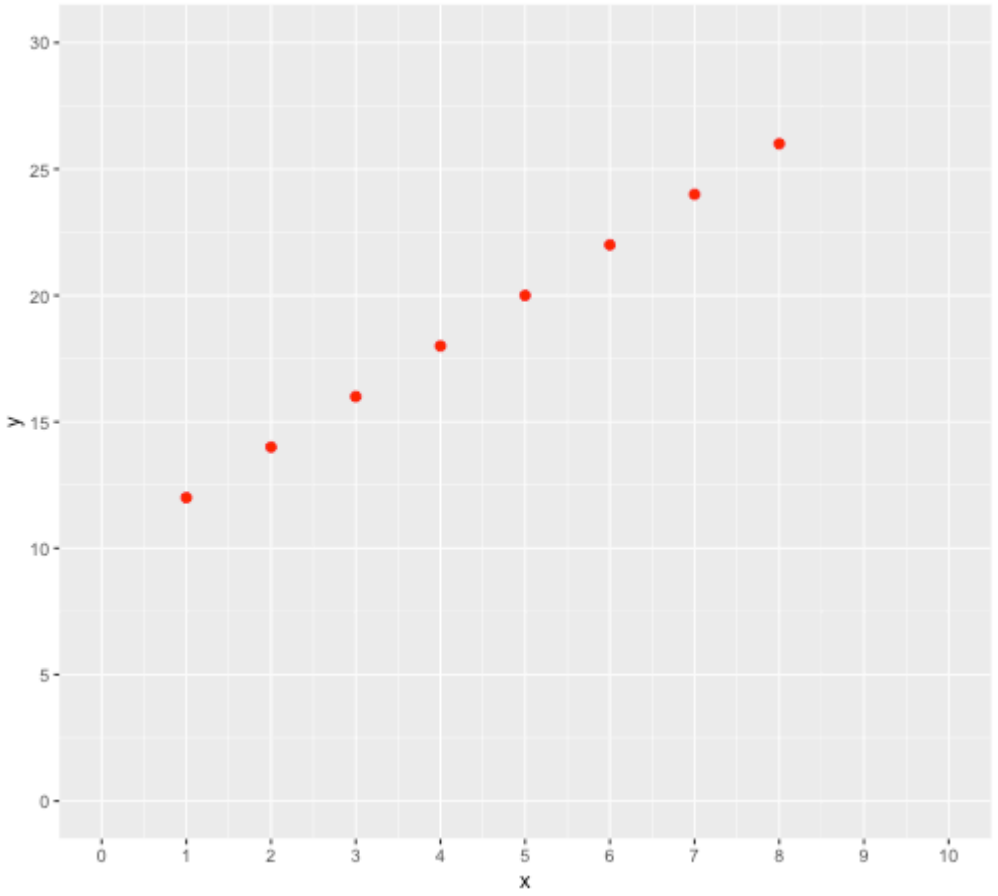
8	26

Visualising Functions



Our Data

x	y
1	12
2	14
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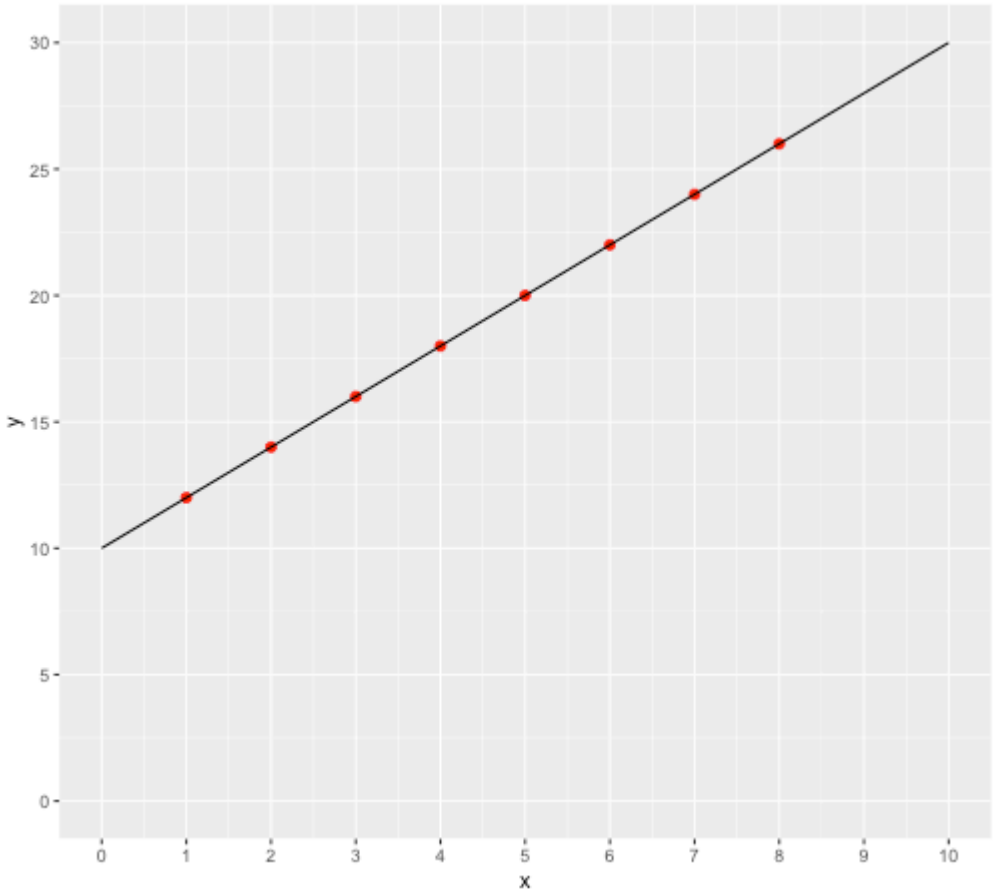
Visualising Functions



Our Data

x	y
1	12
2	14
3	16
4	18
5	20
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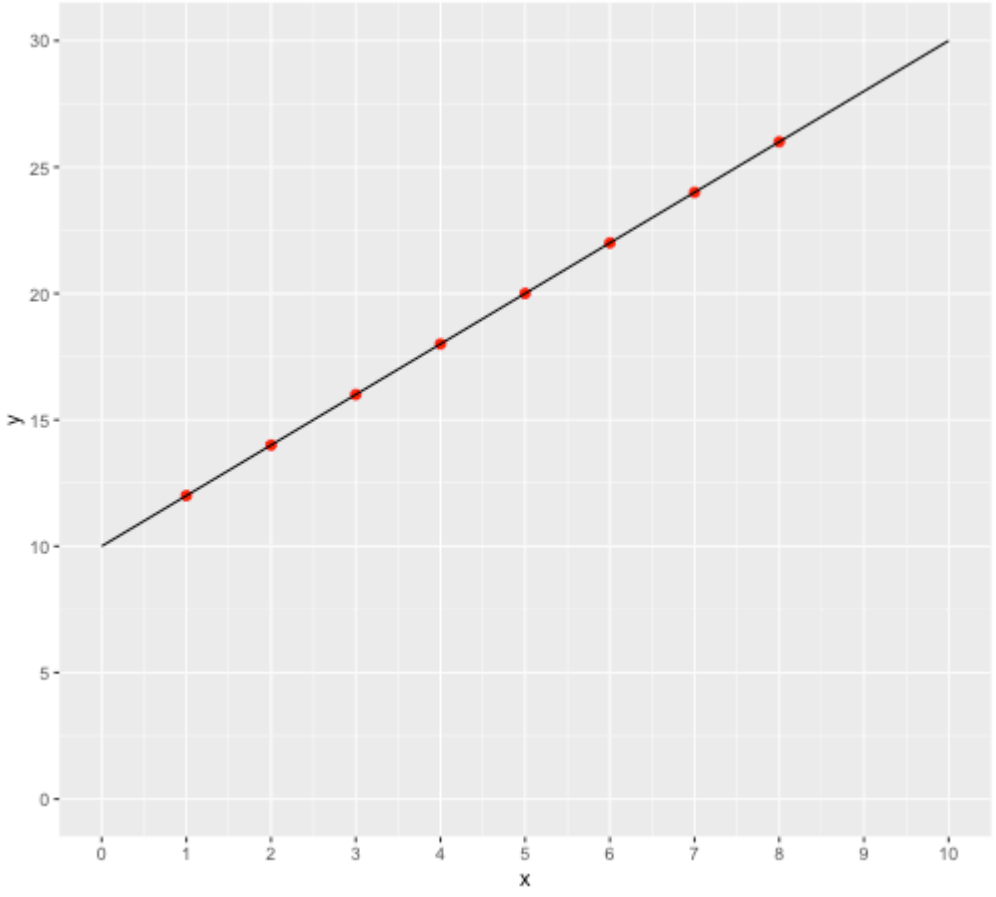
Visualising Functions (R-code)



R-code

```
fun1 <- function(x) 10+(2*x)

ggplot(func_x, aes(x, y)) +
  geom_point(colour = "red", size = 2) +
  stat_function(fun = fun1) +
  scale_x_continuous(name = "x",
                     breaks = c(0:10),
                     labels = c(0:10),
                     limits = c(0,10)) +
  scale_y_continuous(name = "y",
                     breaks = c(seq(0,30,5)),
                     labels = c(seq(0,30,5)),
                     limits = c(0,30))
```

Multiple arguments

- Functions can take multiple arguments. Consider:

$$y = f(x, z) = 10 + (x * z)$$

- Where:

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Multiple arguments

x	z
1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

- Notice that when we have multiple inputs, our rows correspond to pairs of inputs.
- So $x = 1$, pairs with:
  - $z = 1$
  - $z = 2$
  - $z = 3$
- and so on for all values of x

Multiple arguments

x	z	f(x,z)
1	1	11
1	2	12
1	3	13
2	1	12
2	2	14
2	3	16
3	1	13
3	2	16
3	3	19

$$y = f(x, z) = 10 + (x * z)$$

- Example 1, row 2

$$10 + (1 * 2) = 12$$

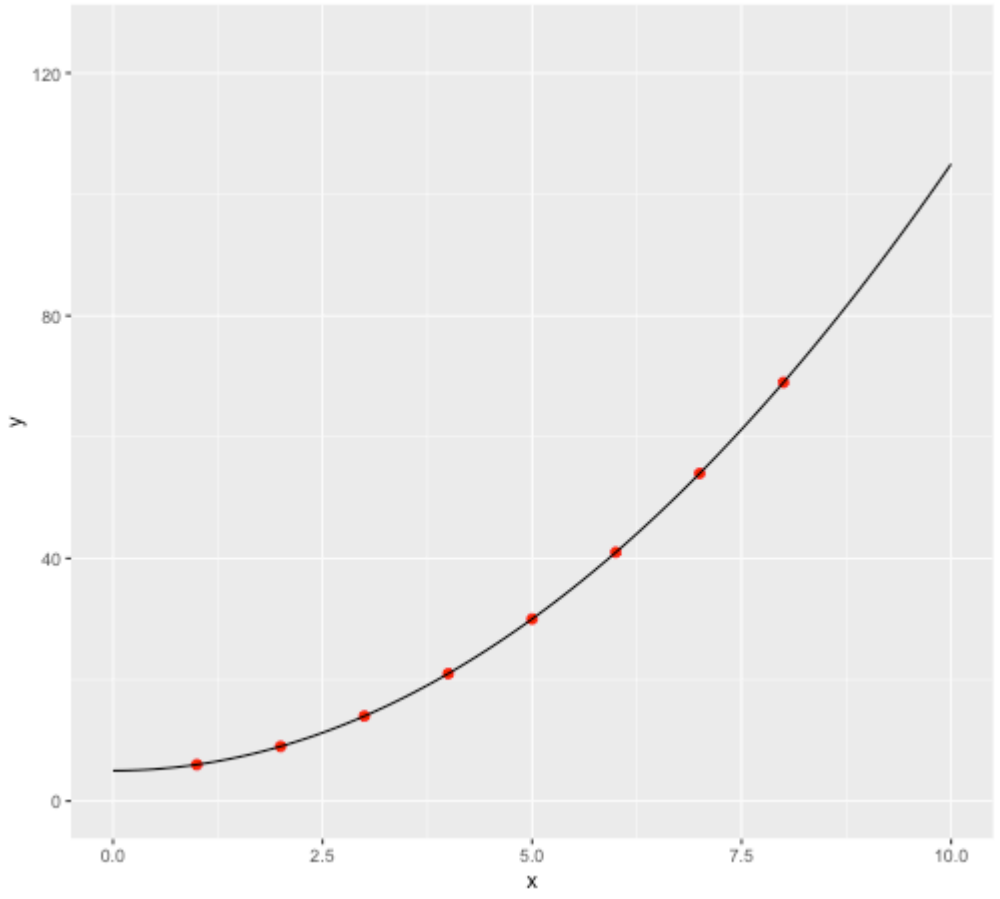
- Example, row 8

$$10 + (3 * 2) = 16$$

Linear vs non-linear functions

- Each of the examples so far have been linear functions.
    - If we plot them, we get a straight line (or flat surface)
- Can also have non-linear functions:
    - A non-linear function would contain powers or roots

Non-linear functions



Example of non-linear function

$$y = f(x) = 5 + x^2$$

Why are functions important?

- There are going to be lots of examples of functions in action.
- Two primary examples are:
    - **Data transformations**
    - **Describing formal models**
- We will start with transformations, and come back to models at the end of the course.

z-scores

- One of the most common transformations in data analysis is standardizing variables.
- What is standardizing?
    - It is putting all variables onto the same scale so they can be compared.
- We refer to standardized variables as z-scores (the reason we will explain later)
- z-score:

$$z = \frac{x - \mu}{\sigma}$$

Z-score for measured variable

- z-score for x:

$$z_{x_i} = \frac{x_i - \bar{x}}{s_x}$$

- Where
  - x_i = individual score on x
  - $\bar{x}$ = mean of x
  - s_x = standard deviation of x

z-scores

- A z-score will have a mean = 0, and a SD = 1.
- What this means is there is a standard way to interpret z-scores.
    - z-score = 1.5, means a respondent is 1.5 SD above the mean.
    - z-score = -2, means a respondent is 2 SD below the mean.

Summary of today

- Functions take input, do something, and produce an output.
- Functions can have multiple arguments, be linear or non-linear
- Typically we will visualize functions
- We use functions frequently in statistics.
- In fact almost everything we are going to see involves functions.

This week

Tasks

- Attend both lectures
- Attend your lab and work together on the lab tasks
- Complete the weekly quiz
    - Opens Monday at 9am
    - Closes Sunday at 5pm
- Submit Formative Report A by 12 noon on Friday the 18th of October 2024 (see instructions in the lab materials)

Support

- **Office hours**: for one-to-one support on course materials or assessments
  (see LEARN > Course information > Course contacts)
- **Piazza**: help each other on this peer-to-peer discussion forum
- **Student Adviser**: for general support while you are at university
  (find your student adviser on MyEd/Euclid)