T-Test: Independent Samples

Data Analysis for Psychology in R 1

Semester 2 Week 7

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Course Overview

Exploratory Data Analysis	Research design and data
	Describing categorical data
	Describing continuous data
	Describing relationships
	Functions
Probability	Probability theory
	Probability rules
	Random variables (discrete)
	Random variables (continuous)
	Sampling

Foundations of inference	Confidence intervals
	Hypothesis testing (p-values)
	Hypothesis testing (critical values)
	Hypothesis testing and confidence intervals
	Errors, power, effect size, assumptions
Common hypothesis tests	One sample t-test
	Independent samples t-test
	Paired samples t-test
	Chi-square tests
	Correlation

Learning Objectives

- Understand when to use an independent samples t-test
- Understand the null hypothesis for an independent sample t-test
- Understand how to calculate the test statistic
- Know how to conduct the test in R

Topics for Today

- Conceptual background and overview of the independent samples t-test
- Independent samples t-test example
- Inferential tests for the independent samples t-test
- Assumptions and effect size

T-Test: Independent Samples

Independent Samples T-Test: Purpose

- The independent t-test is used when we want to test the difference in mean between two measured groups.
- Examples:
    - Treatment versus control group in an experimental study
    - Married versus not married

t-statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{SE_{(\bar{x}_1 - \bar{x}_2)}}$$

- Where
  - $\bar{x}_1$ and $\bar{x}_2$ are the sample means in each group
  - δ_0 is the hypothesised population difference in means in the null hypothesis $(\mu_1 - \mu_2)$
  - $SE_{(\bar{x}_1 - \bar{x}_2)}$ is standard error of the difference
- Sampling distribution is a t-distribution with $n - 2$ degrees of freedom, where n = $n_1 + n_2$

Standard Error Difference

- First calculate the pooled standard deviation

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

- Then use this to calculate the SE of the difference

$$SE_{(\bar{x}_1 - \bar{x}_2)} = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Hypotheses

Two-tailed

$$H_0 : \mu_1 = \mu_2 \qquad \text{vs} \qquad H_1 : \mu_1 \neq \mu_2$$

$$H_0 : \mu_1 - \mu_2 = 0 \qquad \text{vs} \qquad H_1 : \mu_1 - \mu_2 \neq 0$$

One-tailed

$$H_0 : \mu_1 = \mu_2 \qquad \text{vs} \qquad H_1 : \mu_1 < \mu_2$$

$$H_0 : \mu_1 - \mu_2 = 0 \qquad \text{vs} \qquad H_1 : \mu_1 - \mu_2 < 0$$

$$H_0 : \mu_1 = \mu_2 \qquad \text{vs} \qquad H_1 : \mu_1 > \mu_2$$

$$H_0 : \mu_1 - \mu_2 = 0 \qquad \text{vs} \qquad H_1 : \mu_1 - \mu_2 > 0$$

Questions?

Example

Stereotype Threat

- Example taken from Howell, D.C. (2010). *Statistical Methods for Psychology, 7th Edition*. Belmont, CA: Wadsworth Cengage Learning.
- Data from Aronson, Lustina , Good, Keough, Steele and Brown (1998). Experiment on stereotype threat.
    - Two independent groups college students (n=12 control; n=11 threat condition)
    - Both samples excel in maths
    - Threat group told certain students usually do better in the test

Data

```
## # A tibble: 23 × 2
##    Group  Score
##    <fct>  <dbl>
##  1 Threat     7
##  2 Threat     5
##  3 Threat     6
##  4 Threat     5
##  5 Threat     6
##  6 Threat     5
##  7 Threat     4
##  8 Threat     7
##  9 Threat     4
## 10 Threat     3
## # ℹ 13 more rows
```

Hypotheses

- My hypothesis is that the threat group will perform worse than the control group.
- I elect to use a one-tailed test with alpha (α) of .05, and specify the hypotheses as:

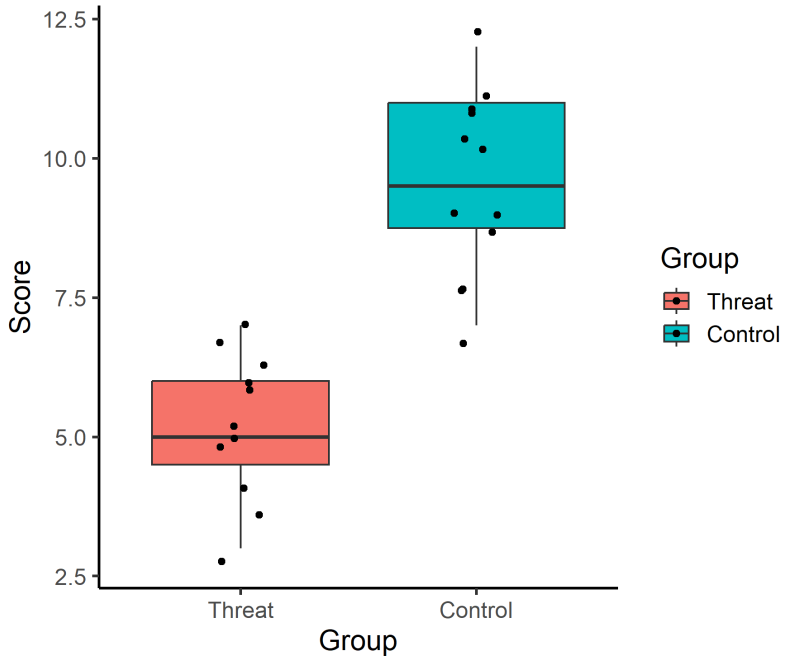
$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

Visualizing Data

- We spoke earlier in the course about the importance of visualizing our data
- Here, we want to show the mean and distribution of scores by group
- So we want a...

Visualizing Data

```
ggplot(data = threat,
       aes(x = Group, y = Score, fill = Group)
  geom_boxplot() +
  geom_jitter(width = 0.1)
```



Calculation

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{SE_{(\bar{x}_1 - \bar{x}_2)}}$$

- Steps to calculate t:
  - Calculate the sample mean in both groups $\bar{x}_1$ and $\bar{x}_2$
  - Calculate the pooled SD (s_p)
  - Check I know my n
  - Calculate the standard error (SE)

Calculation

```
threat |>
  group_by(Group) |>
  summarise(
    Mean = mean(Score),
    SD = sd(Score),
    n = n()
  ) |>
  kable(digits = 2) |>
  kable_styling(full_width = FALSE)
```

Group	Mean	SD	n
Threat	5.27	1.27	11
Control	9.58	1.51	12

Calculation

Group	Mean	SD	n
Threat	5.27	1.27	11
Control	9.58	1.51	12

- Calculate pooled standard deviation:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(11 - 1) \cdot 1.27^2 + (12 - 1) \cdot 1.51^2}{11 + 12 - 2}} = \sqrt{\frac{10 \cdot 1.27^2 + 11 \cdot 1.51^2}{11 + 12 - 2}} = \sqrt{\frac{41.21}{21}} = 1.401$$

- Calculate the standard error:

$$SE_{(\bar{x}_1 - \bar{x}_2)} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 1.401 \cdot \sqrt{\frac{1}{11} + \frac{1}{12}} = 1.401 \cdot 0.417 = 0.584$$

Calculation

- Steps in my calculations:
  - Calculate the sample mean in both groups - Threat $(\bar{x}_1 = 5.27)$, Control $(\bar{x}_2 = 9.58)$
  - Calculate the pooled SD $(s_p = 1.401)$
  - Check I know my n - Threat $(n_1 = 11)$ and Control $(n_2 = 12)$ - $n = 23$
  - Calculate the standard error $(SE = 0.584)$.
- Use all this to calculate t

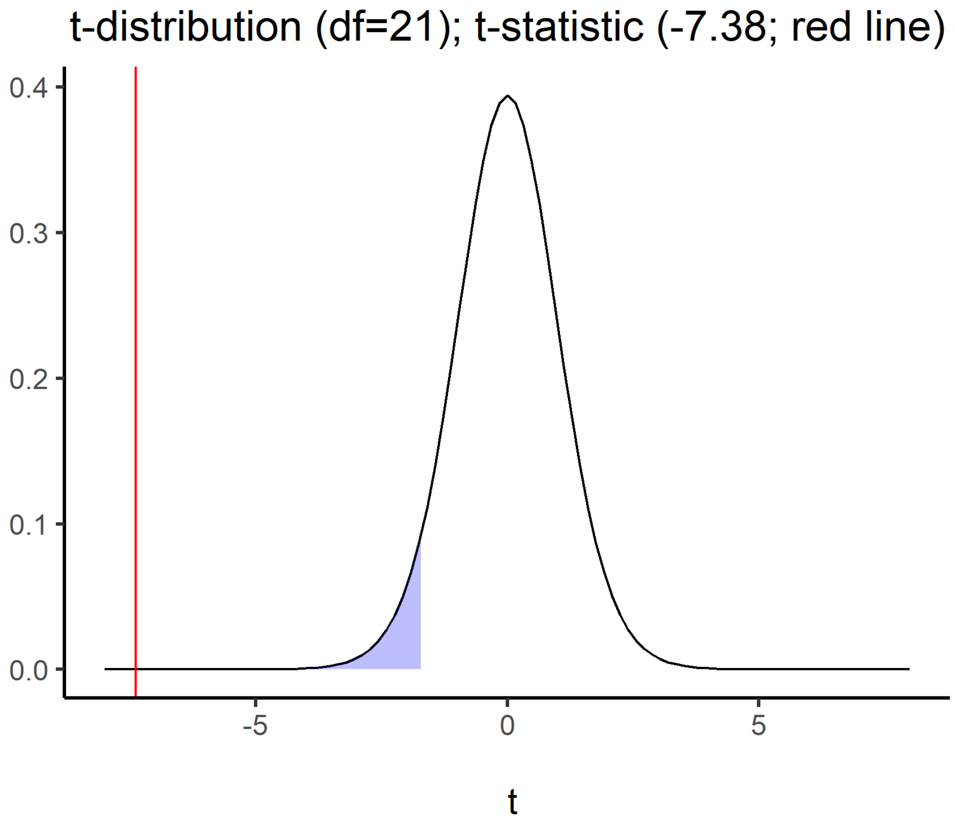
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{SE_{(\bar{x}_1 - \bar{x}_2)}} = \frac{5.27 - 9.58}{0.584} = -7.38$$

- So in our example $t = -7.38$
- Note: When doing hand calculations there might be a small amount of rounding error when we compare to t calculated in R

Is our Test Significant?

- We have all the pieces we need:
    - Degrees of freedom = $n - 2 = (12 + 11) - 2 = 23 - 2 = 21$
    - We have our t-statistic (-7.38)
    - Hypothesis to test (one-tailed)
    - α level (.05)
- Now all we need is the critical value from the associated t-distribution in order to make our decision

Is our Test Significant?




```
tibble(
  LowerCrit = round(qt(0.05, 21),2),
  Exactp = 1-pt(7.3817, 21)
)
```

```
## # A tibble: 1 × 2
##   LowerCrit      Exactp
##       <dbl>       <dbl>
## 1     -1.72 0.000000146
```

Is our Test Significant?

- The critical value is -1.72, and our t-statistic (-7.38) is larger than this
- We found that $p < .001$, which is $< \alpha$
- Thus, we **reject the null hypothesis**

Independent Samples T-Test **in R**

```
res <- t.test(threat$Score ~ threat$Group,
       alternative = "less",
       mu = 0,
       var.equal = TRUE,
       conf.level = 0.95)
res
```

```
##
##      Two Sample t-test
##
## data:  threat$Score by threat$Group
## t = -7.3817, df = 21, p-value = 1.458e-07
## alternative hypothesis: true difference in means between group Threat and group Control is less than 0
## 95 percent confidence interval:
##       -Inf -3.305768
## sample estimates:
##  mean in group Threat mean in group Control
##              5.272727              9.583333
```

To get **missing** CI - need to do a **two-sided** test

```
t.test(threat$Score ~ threat$Group,
       alternative = "two.sided",
       mu = 0,
       var.equal = TRUE,
       conf.level = 0.95)
```

Write Up

An independent samples t-test was used to determine whether the average maths score of the stereotype threat group $(n = 11)$ was significantly lower $(\alpha = .05)$ than the control group $(n = 12)$. There was a significant difference in test score between the control $(M = 9.58, SD = 1.51)$ and threat $(M = 5.27, SD = 1.27)$ groups, where the scores were significantly lower in the threat group $(t(21) = -7.38, p < .001, one - tailed)$. Therefore, we can reject the null hypothesis. The direction of difference supports our directional hypothesis and indicates that the threat group performed more poorly than the control group.

Questions?

Data Requirements & Assumptions

Data Requirements

- A numeric variable
- A binary variable denoting groups

Assumption Checks Summary

	Description	One-Sample t-test	Independent Samples t-test	Paired Samples t-test
Normality	Numeric variable (or difference) is normally distributed OR sample size is sufficiently large.	Yes (variable). Sample size guideline: $n \geq 30$	Yes (variable in each group). Sample size guideline: $n1 \geq 30$ and $n2 \geq 30$	Yes (difference). Sample size guideline: number of pairs ≥ 30
Tests:	Descriptive Statistics and Plots; QQ-Plot; Shapiro-Wilks Test			
Independence	Observations are sampled independently.	Yes	Yes (within and across groups)	Yes (across pairs)
Tests:	None. Design issue.			
Homogeneity of variance	Population standard deviation is the same in both groups.	NA	Yes*	NA
Tests:	F-test			
Matched Pairs in data	For paired sample, each observation must have matched pair.	NA	NA	Yes
Tests:	None. Data structure issue.			

* Welch t-test is available if this is not met

Data Requirements & Assumptions: How to Check/Test

- DV is numeric
  - The dependent variable should be measured on a interval/ratio/integer scale
- Normality **within groups**
  - Can be checked with descriptive statistics, visually with plots, and with a Shapiro-Wilks test for each group separately
- Independence of observations **within and across groups**
  - More of a study design issue, and cannot directly test
  - Need to make sure that each individual only belongs to one group, and only has one observation in the group they belong to
- Homogeneity of variance **across groups**
  - Can be checked using an F-test

Normality: Skew

- Skew is a descriptive statistic informing us of both the direction and magnitude of asymmetry
  - Below are some rough guidelines on how to interpret skew
  - No strict cuts for skew - these are loose guidelines

Verbal label	Magnitude of skew in absolute value
Generally not problematic	$\lvert \text{Skew} \rvert < 1$
Slight concern	$1 > \lvert \text{Skew} \rvert < 2$
Investigate impact	$\lvert \text{Skew} \rvert > 2$

Skew in R

```r
library(psych)
threat |>
  group_by(Group) |>
    summarise(
      skew = round(skew(Score),2)
  )
```

```
## # A tibble: 2 × 2
##   Group    skew
##   <fct>   <dbl>
## 1 Threat  -0.2
## 2 Control -0.07
```

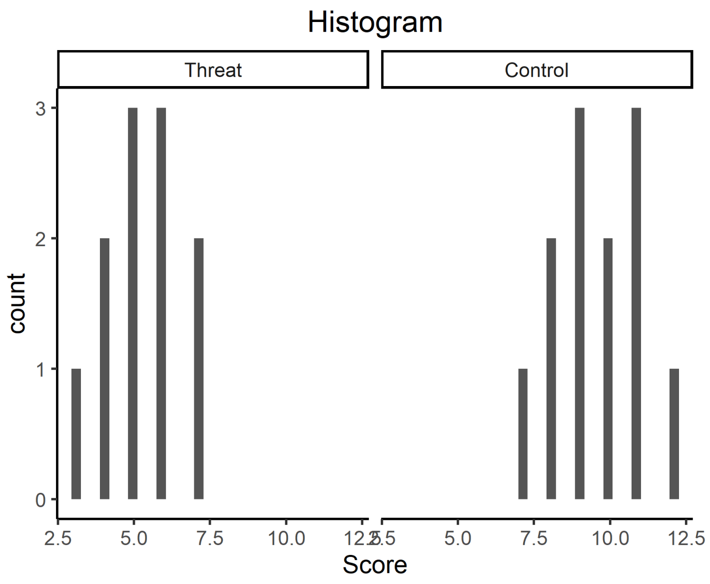
Normality: Visual Assessment

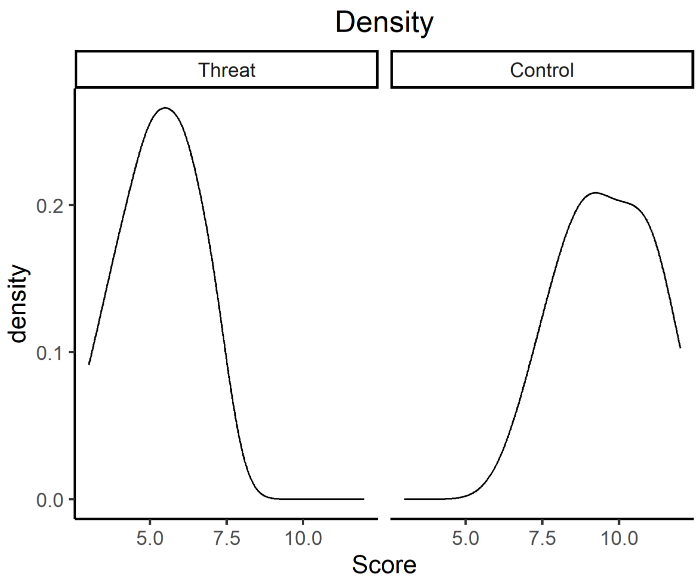
- We can visually assess normality by plotting the distribution of our outcome variable in both groups separately:
    - Histograms
        - The count (or frequency) of data points that fall within specified intervals/bins
    - Density Plots
        - The probability density (or proportion of values) of data points at each value of the observed variable
    - QQ-Plots (Quantile-Quantile plot):
        - Plots the sorted quantiles of one data set (distribution) against sorted quantiles of data set (distribution)
        - Quantile = the percent of points falling below a given value
        - For a normality check, we can compare our own data to data drawn from a normal distribution

Histogram & Density Plots **in R**

```
ggplot(data = threat, aes(x=Score)) +
  geom_histogram() +
  facet_wrap(~ Group) +
  labs(title = "Histogram")
```

```
ggplot(data = threat, aes(x=Score)) +
  geom_density() +
  facet_wrap(~ Group) +
  labs(title = "Density")
```

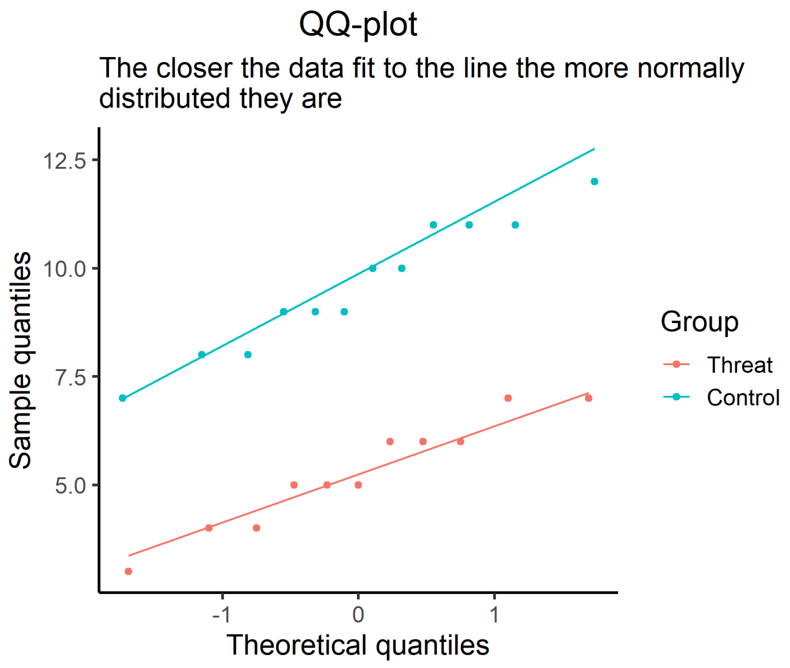




- No concerns in histogram or density plots for either group

QQ-Plots **in R**

```
ggplot(data = threat,
       aes(sample = Score, colour = Group)) +
  geom_qq() +
  geom_qq_line() +
      labs(title="QQ-plot",
       x = "Theoretical quantiles",
       y = "Sample quantiles")
```



- This looks reasonable in both groups

Normality: Shapiro-Wilks Test

- Shapiro-Wilks test:
  - Checks properties of the observed data against properties we would expected from normally distributed data.
  - Statistical test of normality.
  - H_0: data = a normal distribution.
  - p-value $< \alpha$ = reject the null, data are not normal.
    - Sensitive to n as all p-values will be.
    - In very large n, normality should also be checked with QQ-plots alongside statistical test.

Shapiro-Wilks Test **in R**

```
threat |>
  filter(Group == "Control") |>
  pull(Score) |>
shapiro.test()
```

```
##
##      Shapiro-Wilk normality test
##
## data:  pull(filter(threat, Group == "Control"), Score)
## W = 0.95538, p-value = 0.7164
```

$W = 0.96, p = .716$

```
thr <- threat |>
  filter(Group == "Threat") |>
  select(Score)
shapiro.test(thr$Score)
```

```
##
##      Shapiro-Wilk normality test
##
## data:  thr$Score
## W = 0.93979, p-value = 0.518
```

$W = 0.94, p = .518$

Homogeneity of Variance: F-Test

- The F-test is a test that compares the variances of two groups
  - This test is preferable for t-test
- Hypotheses:
  - H_0: Population variances are equal
  - H_1: Population variances are **not** equal
- Interpretation:
  - If p-value $< \alpha$, then reject the null as the variances differ across groups

F-test in R

```
var.test(threat$Score ~ threat$Group, ratio = 1)
```

```
##
##      F test to compare two variances
##
## data:  threat$Score by threat$Group
## F = 0.71438, num df = 10, denom df = 11, p-value = 0.6038
## alternative hypothesis: true ratio of variances is not equal to 1
## 95 percent confidence interval:
##  0.2026227 2.6181459
## sample estimates:
## ratio of variances
##          0.7143813
```

Why `ratio = 1`?

$$H_0: \quad \sigma_1^2 = \sigma_2^2 \quad \text{which is equivalent to} \quad \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_1: \quad \sigma_1^2 \neq \sigma_2^2 \quad \text{which is equivalent to} \quad \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

Violation of Homogeneity of Variance

- If the variances differ, we can use a Welch test.
- Conceptually very similar, but we do not use a pooled standard deviation.
    - As such our estimate of the SE of the difference changes
    - As do our degrees of freedom

Welch Test

- Test statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{SE_{(\bar{x}_1 - \bar{x}_2)}}$$

- SE calculation:

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- Degrees of freedom:

$$df = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{(\frac{s_1^2}{n_1})^2}{n_1 - 1} + \frac{(\frac{s_2^2}{n_2})^2}{n_2 - 1}}$$

Welch Test **in R**

```r
t.test(threat$Score ~ threat$Group,
       alternative = "less",
       mu = 0,
       var.equal = FALSE, #default, only here to highlight difference
       conf.level = 0.95)
```

```
##
##     Welch Two Sample t-test
##
## data:  threat$Score by threat$Group
## t = -7.4379, df = 20.878, p-value = 1.346e-07
## alternative hypothesis: true difference in means between group Threat and group Control is less than 0
## 95 percent confidence interval:
##       -Inf -3.313093
## sample estimates:
##  mean in group Threat mean in group Control
##              5.272727              9.583333
```

Questions?

Effect Size

Cohen's D: Independent Samples T-Test

If you **do** have equality of variances:

$$D = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{s_p}$$

- $\bar{x}_1$ = mean group 1
- $\bar{x}_2$ = mean group 2
- δ_0 = the hypothesised population difference in means in the null hypothesis $(\mu_1 - \mu_2)$
- s_p = pooled standard deviation

If you **do not** have equality of variances:

- Calculate via `cohens_d()` function from **`effectsize`** package in R - do not calculate by hand

Recall the common "cut-offs" for D-scores:

Verbal label	**Magnitude of D in absolute value**
Small (or weak)	≤ 0.20
Medium (or moderate)	≈ 0.50
Large (or strong)	≥ 0.80

Cohen's D: Independent Samples T-Test **in R**

```r
library(effectsize)
cohens_d(threat$Score ~ threat$Group,
         mu = 0,
         alternative = "less",
         var.equal = TRUE,
         ci = 0.95)
```

```
## Cohen's d |         95% CI
## -------------------------
## -3.08     | [-Inf, -2.02]
##
## - Estimated using pooled SD.
## - One-sided CIs: lower bound fixed at [-Inf].
```

To get **missing** CI - need to do a **two-sided** test:

```r
cohens_d(threat$Score ~ threat$Group,
       alternative = "two.sided",
       mu = 0,
       var.equal = TRUE,
       ci = 0.95)
```

```
## Cohen's d |         95% CI
## --------------------------
## -3.08     | [-4.30, -1.83]
##
## - Estimated using pooled SD.
```

Write Up: Data Requirements, Assumptions, & Effect Size

The DV of our study, Score, was measured on a continuous scale, and data were independent (participants belonged to one of two groups - Control or Threat). The assumption of normality was visually assessed (via histograms, density plots, and a QQplot) as well as statistically via a Shapiro-Wilks test. The QQplots did not show much deviation from the diagonal line in either group, and the Shapiro-Wilks test for both the Control $(W = 0.96, p = .716)$ and Threat $(W = 0.94, p = .518)$ conditions suggested that the samples came from a population that was normally distributed. This was inline with the histogram and density plots for each group, which suggested that Score was normally distributed (and where $skew < 1$). Based on the results of our F-test, there was no significant difference between the two population variances $(F(10, 11) = 0.71, p = .604)$. The size of the effect was found to be large $D = -3.08\ [-4.30, -2.02]$.

Summary

- Today we have covered:
  - Basic structure of the independent-sample t-test
  - Calculations
  - Interpretation
  - Assumption checks
  - Effect size measures

This Week

Tasks

- Attend both lectures
- Attend your lab and work on the assessed report with your group (due by 12 noon on Friday 28th of March 2025)
- Complete the weekly quiz
    - Opened Monday at 9am
    - Closes Sunday at 5pm

Support

- **Office Hours**: for one-to-one support on course materials or assessments
(see LEARN > Course information > Course contacts)
- **Piazza**: help each other on this peer-to-peer discussion forum
- **Student Adviser**: for general support while you are at university
(find your student adviser on MyEd/Euclid)