# Correlation

## Data Analysis for Psychology in R 1

**Dr Emma Waterston**

Department of Psychology
The University of Edinburgh

# Week's Learning Objectives

1. Understand the difference between variance, covariance, and correlation
2. Know how to calculate both covariance and correlation
3. Understand how to interpret the correlation coefficient
4. Know how perform and interpret the results of a significance test of your correlation
5. Understand which form of correlation is most appropriate to use with your data

# Example

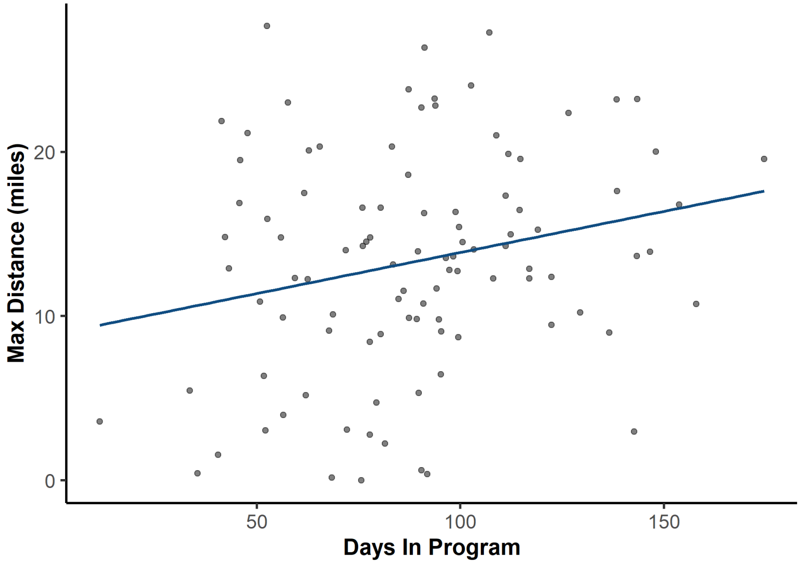
- Suppose a running coach wants to know whether there is an association between a participants' time spent enrolled in their running program (recorded in days) and the maximum distance they can run (receded in miles)
- We are provided with information on the last 100 participants to enroll on the program

# Data

```
##   daysInProgram maxDistance
## 1      95.32143    9.076514
## 2     174.62360   19.581428
## 3      11.38827    3.575377
## 4      91.17004   26.352013
## 5      56.37932    9.923332
## 6     122.36747   12.384450
```

# Visualisation

```
ggplot(dat, aes(daysInProgram, maxDistance)) +
  geom_point() +
  geom_smooth(method = 'lm', se=FALSE) +
  labs(x='Days In Program',
       y='Max Distance (miles)',
       title = "Association between Running Di
```



# Part 1: Variance & Covariance

# Variance Recap

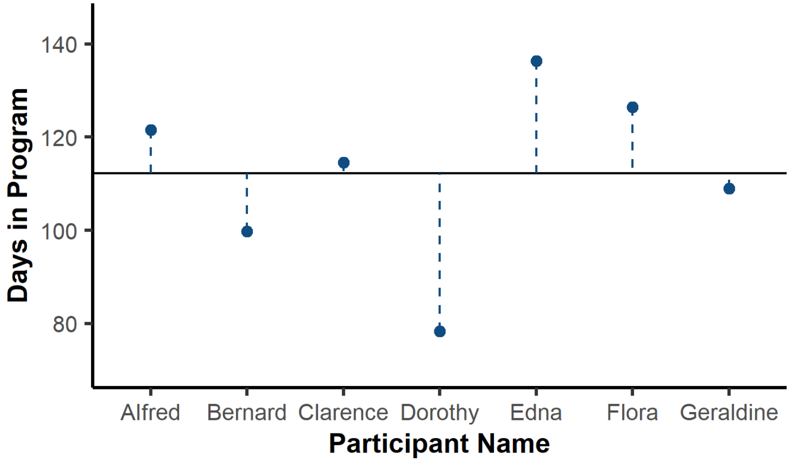
$$s_x^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

- **Variance:** Deviance around the mean of a single variable

- Raw deviation is the distance between each person's days in the program and the mean number of days in the program.

- To get the variance, we:

    1. Square the values to get rid of the negative
    2. Sum them up and divide by $n-1$ to get the average deviation of the group from its mean.



# Variance

$$s_x^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} = s_x^2 = \frac{2192.57}{7-1} = s_x^2 = 365.43$$

- where $\bar{x} = 112.3$

| $x_i$ | $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ |
|---|---|---|
| 121.57 | 9.27 | 86.01 |
| 99.73 | -12.57 | 157.9 |
| 114.63 | 2.33 | 5.45 |
| 78.37 | -33.93 | 1150.95 |
| 136.41 | 24.11 | 581.5 |
| 126.42 | 14.12 | 199.5 |
| 108.94 | -3.36 | 11.26 |
| | | **2192.57** |

# Covariance Recap

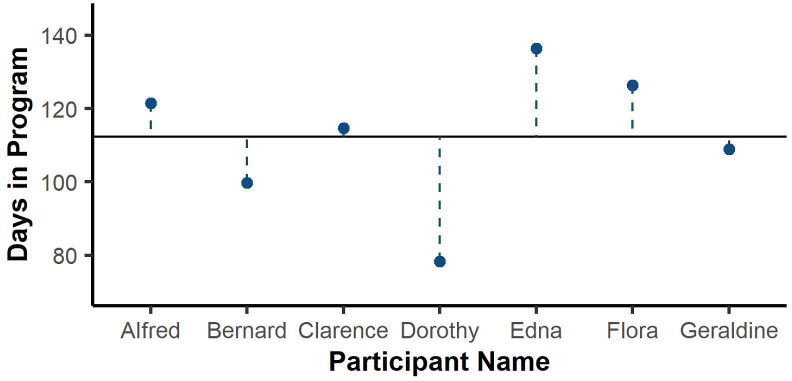
- **Covariance:** A value that represents how two variables change together
- Does $y$ differ from its mean in a similar way to $x$?
- Mathematically similar to variance:

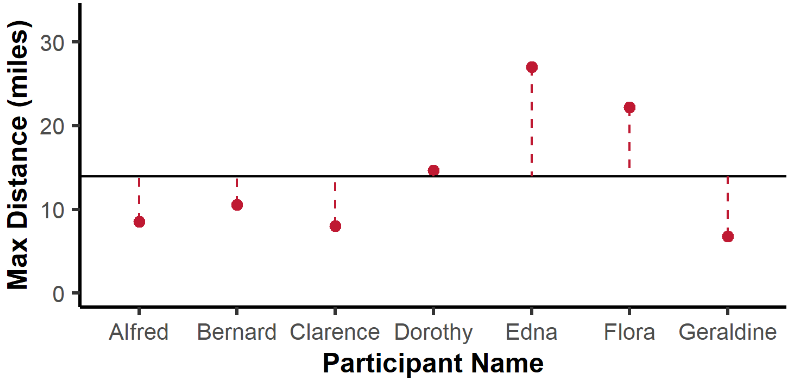
### Variance

$$s_x^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})}{n-1}$$

### Covariance

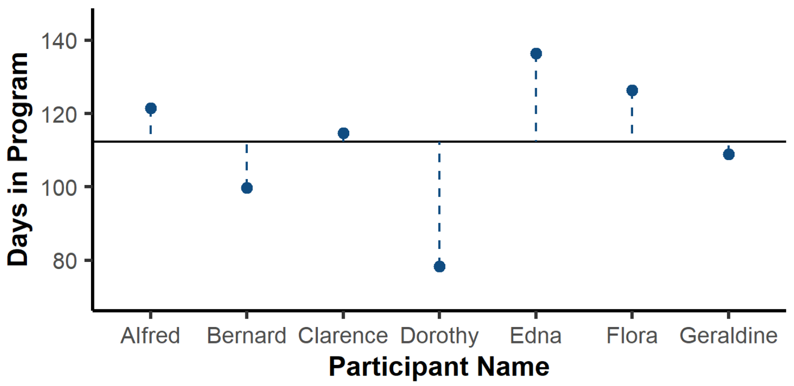
$$Cov_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

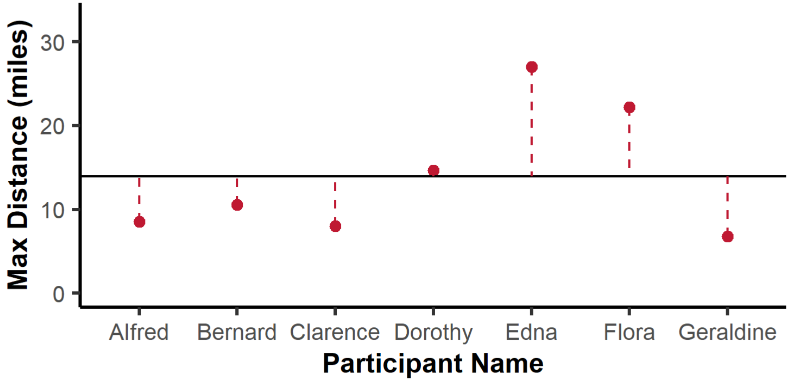


# Covariance Recap

- It's possible two variables are related if their observations differ proportionally from their means in a consistent way
- Covariance gives us a sense of this...
    - High covariance suggests a stronger association than a lower covariance
    - Why can't we stop here?
    - Why is correlation necessary?

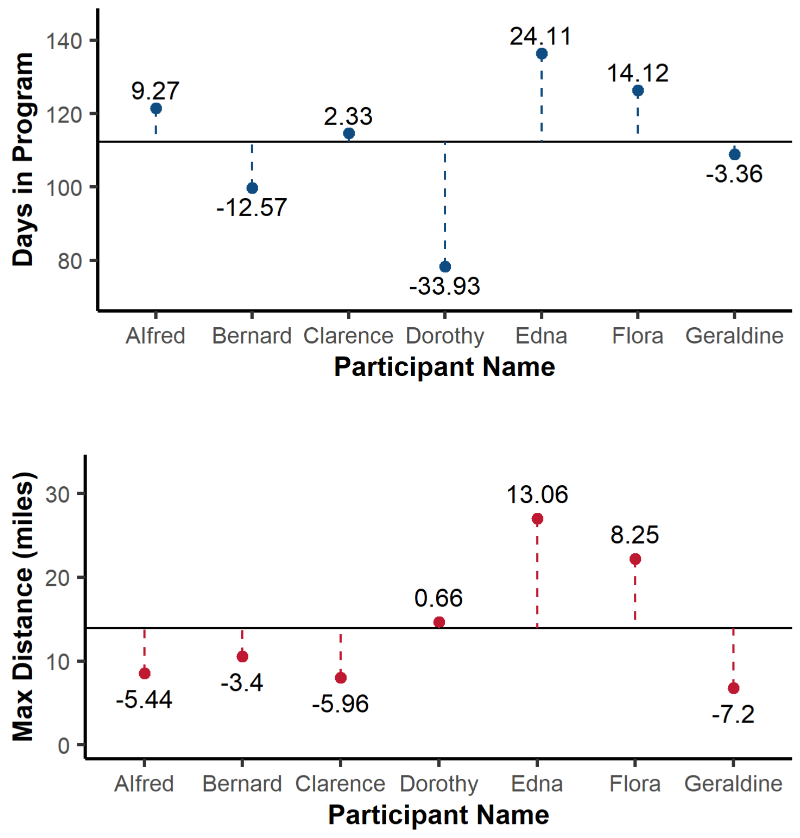


# The Trouble with Covariance

$$Cov_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

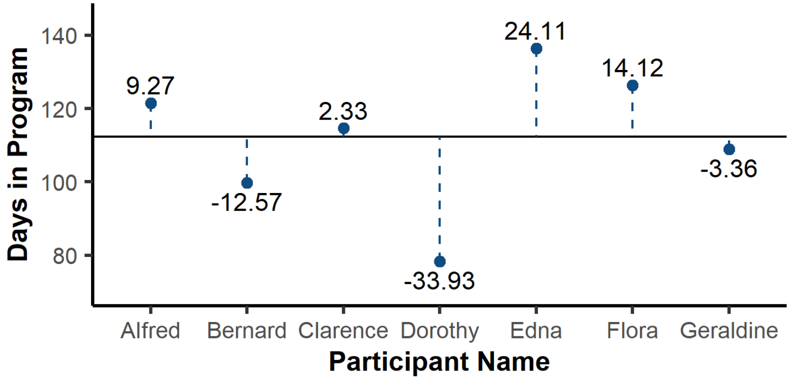
| $x_i - \bar{x}$ | $y_i - \bar{y}$ | $(x_i - \bar{x})(y_i - \bar{y})$ |
|---|---|---|
| 9.27 | -5.44 | -50.41 |
| -12.57 | -3.4 | 42.67 |
| 2.33 | -5.96 | -13.9 |
| -33.93 | 0.66 | -22.54 |
| 24.11 | 13.06 | 315.04 |
| 14.12 | 8.25 | 116.59 |
| -3.36 | -7.2 | 24.15 |
| | | **411.59** |

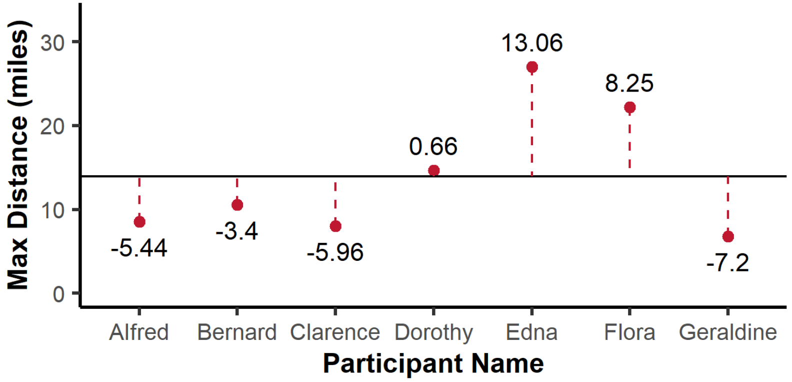


# The Trouble with Covariance

$$Cov_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{411.59}{7-1} = 68.6$$

| $x_i - \bar{x}$ | $y_i - \bar{y}$ | $(x_i - \bar{x})(y_i - \bar{y})$ |
|---|---|---|
| 9.27 | -5.44 | -50.41 |
| -12.57 | -3.4 | 42.67 |
| 2.33 | -5.96 | -13.9 |
| -33.93 | 0.66 | -22.54 |
| 24.11 | 13.06 | 315.04 |
| 14.12 | 8.25 | 116.59 |
| -3.36 | -7.2 | 24.15 |
| | | **411.59** |



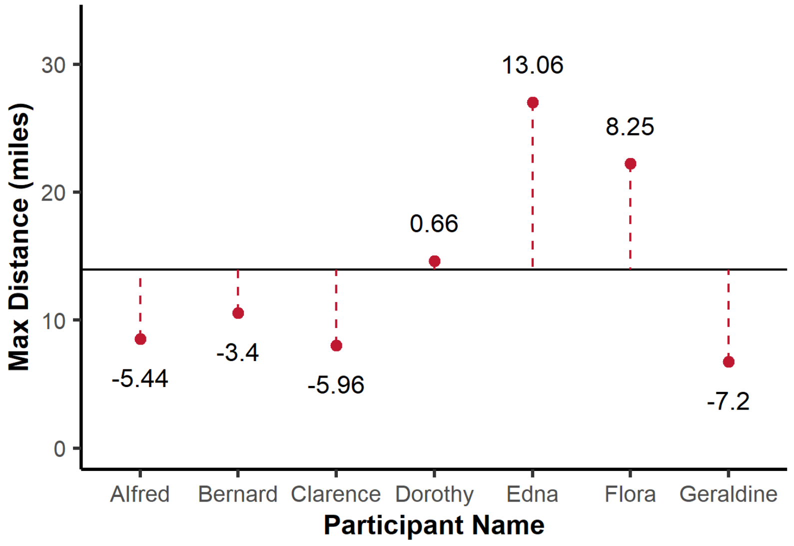
# The Trouble with Covariance

- A value of 68.6 seems high...I think. Is it?
  - Maybe? But maybe not.
  - Covariance is related specifically to the scales of the variables we are analysing.
  - Variables with larger scales will naturally have larger covariance values.

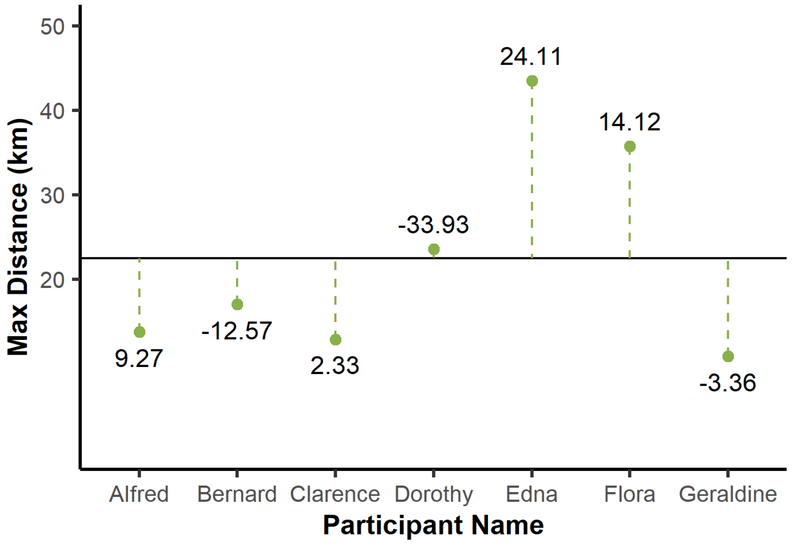
# The Trouble with Covariance

- Consider what would happen if we converted our distance data to kilometers instead of miles.

**Miles**




**Kilometers**



# The Trouble with Covariance

**Miles**

$$Cov_{xy} = \frac{411.59}{7-1} = 68.6$$

| $x_i - \bar{x}$ | $y_i - \bar{y}$ | $(x_i - \bar{x})(y_i - \bar{y})$ |
|---|---|---|
| 9.27 | -5.44 | -50.41 |
| -12.57 | -3.4 | 42.67 |
| 2.33 | -5.96 | -13.9 |
| -33.93 | 0.66 | -22.54 |
| 24.11 | 13.06 | 315.04 |
| 14.12 | 8.25 | 116.59 |
| -3.36 | -7.2 | 24.15 |
| | | **411.59** |

**Kilometers**

$$Cov_{xy} = \frac{662.66}{7-1} = 110.44$$

| $x_i - \bar{x}$ | $y_i - \bar{y}$ | $(x_i - \bar{x})(y_i - \bar{y})$ |
|---|---|---|
| 9.27 | -8.75 | -81.16 |
| -12.57 | -5.47 | 68.7 |
| 2.33 | -9.59 | -22.38 |
| -33.93 | 1.07 | -36.28 |
| 24.11 | 21.03 | 507.21 |
| 14.12 | 13.29 | 187.7 |
| -3.36 | -11.59 | 38.88 |
| | | **662.66** |

# Part 2: The Correlation Coefficient

# Correlation

- Correlation allows you to compare continuous variables across different scales without the magnitude of the variables skewing your results.

- **Pearson's product moment correlation**, $r$, is the standardised version of covariance:

$$r = \frac{\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}\sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}}} = \frac{Cov_{xy}}{s_x s_y}$$

- By dividing covariance by the product of the standard deviations of $x$ and $y$, we remove issues with scale differences in the original variables.

- Because of this, you can use $r$ to investigate the association(s) between continuous variables with completely different ranges.

# Correlation

**Miles**

| $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ | $y_i - \bar{y}$ | $(y_i - \bar{y})^2$ |
|---|---|---|---|
| 9.27 | 86.01 | -5.44 | 29.55 |
| -12.57 | 157.9 | -3.4 | 11.53 |
| 2.33 | 5.45 | -5.96 | 35.47 |
| -33.93 | 1150.95 | 0.66 | 0.44 |
| 24.11 | 581.5 | 13.06 | 170.68 |
| 14.12 | 199.5 | 8.25 | 68.13 |
| -3.36 | 11.26 | -7.2 | 51.78 |
| | **2192.57** | | **367.58** |

**Kilometers**

| $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ | $y_i - \bar{y}$ | $(y_i - \bar{y})^2$ |
|---|---|---|---|
| 9.27 | 86.01 | -8.75 | 76.59 |
| -12.57 | 157.9 | -5.47 | 29.89 |
| 2.33 | 5.45 | -9.59 | 91.94 |
| -33.93 | 1150.95 | 1.07 | 1.14 |
| 24.11 | 581.5 | 21.03 | 442.41 |
| 14.12 | 199.5 | 13.29 | 176.61 |
| -3.36 | 11.26 | -11.59 | 134.21 |
| | **2192.57** | | **952.8** |

$$s_x = \sqrt{\frac{2192.57}{7-1}} = 19.12 \quad s_y = \sqrt{\frac{367.58}{7-1}} = 7.83 \quad s_x = \sqrt{\frac{2192.57}{7-1}} = 19.12 \quad s_y = \sqrt{\frac{952.8}{7-1}} = 12.6$$

# Correlation

**Miles**

| $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ | $y_i - \bar{y}$ | $(y_i - \bar{y})^2$ |
|---|---|---|---|
| 9.27 | 86.01 | -5.44 | 29.55 |
| -12.57 | 157.9 | -3.4 | 11.53 |
| 2.33 | 5.45 | -5.96 | 35.47 |
| -33.93 | 1150.95 | 0.66 | 0.44 |
| 24.11 | 581.5 | 13.06 | 170.68 |
| 14.12 | 199.5 | 8.25 | 68.13 |
| -3.36 | 11.26 | -7.2 | 51.78 |
| | **2192.57** | | **367.58** |

$$r = \frac{Cov_{xy}}{s_x s_y} = \frac{68.6}{19.12 \cdot 7.83} = 0.46$$

**Kilometers**

| $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ | $y_i - \bar{y}$ | $(y_i - \bar{y})^2$ |
|---|---|---|---|
| 9.27 | 86.01 | -8.75 | 76.59 |
| -12.57 | 157.9 | -5.47 | 29.89 |
| 2.33 | 5.45 | -9.59 | 91.94 |
| -33.93 | 1150.95 | 1.07 | 1.14 |
| 24.11 | 581.5 | 21.03 | 442.41 |
| 14.12 | 199.5 | 13.29 | 176.61 |
| -3.36 | 11.26 | -11.59 | 134.21 |
| | **2192.57** | | **952.8** |

$$r = \frac{Cov_{xy}}{s_x s_y} = \frac{110.44}{19.12 \cdot 12.6} = 0.46$$

# Correlations

- Correlations measure the degree of association between two variables.
- If one variable changes, does the other variable also change?
- If so, do they rise and fall together, or does one rise as the other falls?

# Correlation in R

- To run a simple correlation in R, you can use `cor()`
- Let's compute the correlation between the number of days in the program and the max running distance for our entire sample of 100:

```
cor(dat$daysInProgram, dat$maxDistance)
```

```
## [1] 0.2332039
```

- Now we have a correlation value. But what does it mean?

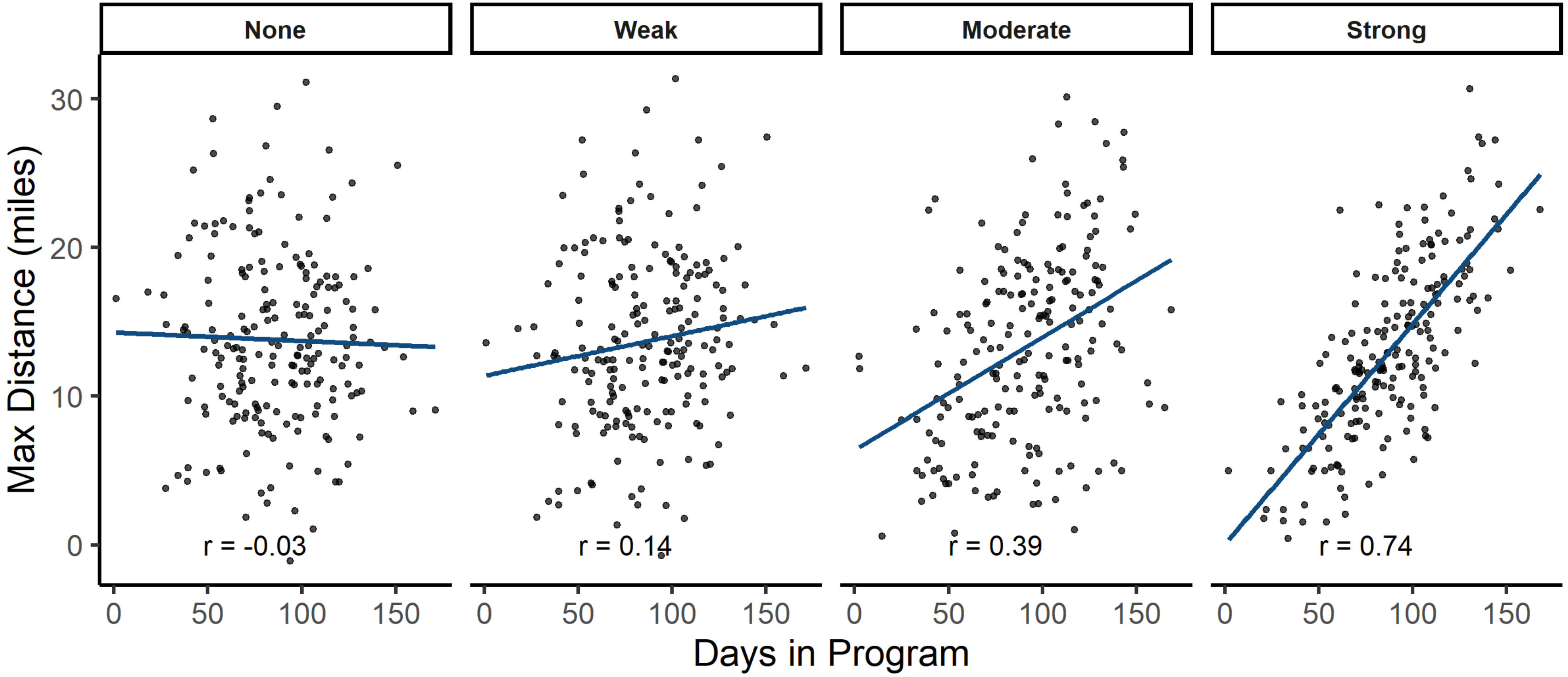
# Interpreting $r$

- Values of $r$ fall between -1 and 1.
  - Values closer to 0 indicate a weaker association
  - More extreme values indicate a stronger association
  - Interpretation:

| Strength | Value |
|---|---|
| Weak | $.1 < |r| < .3$ |
| Moderate | $.3 < |r| < .5$ |
| Strong | $|r| > .5$ |

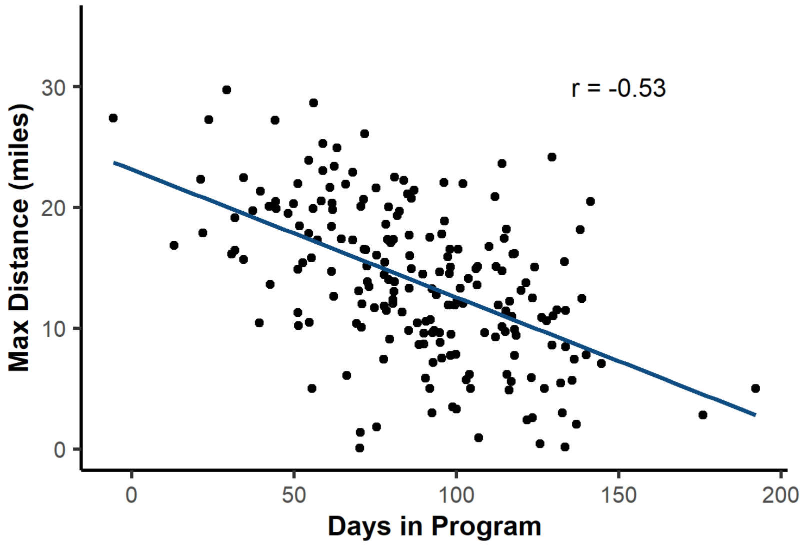
# Interpreting $r$

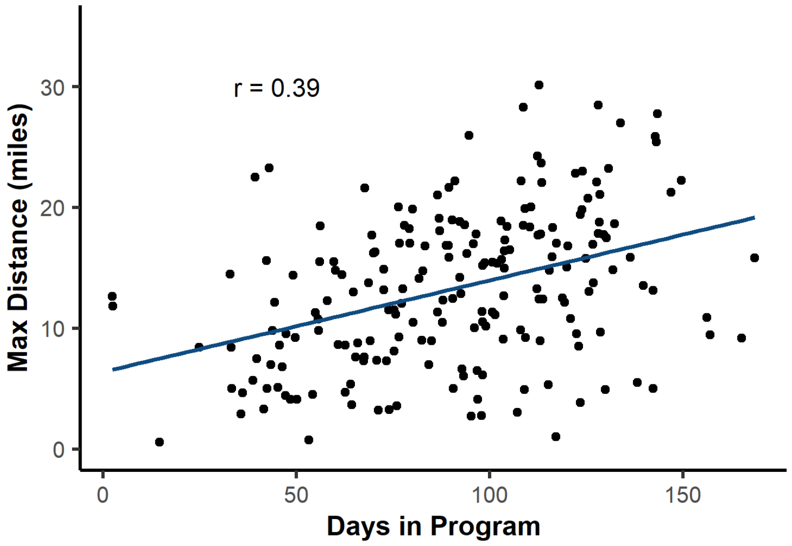
- Values of $r$ fall between -1 and 1.
  - Values closer to 0 indicate a weaker association
  - More extreme values indicate a stronger association



# Interpreting $r$

- The sign of $r$ says nothing about the strength of the association, but its direction
  - Positive values indicate that the two variables rise together or fall together.
  - Negative values indicate that as one variable increases, the other decreases, and vice versa



# Interpreting $r$

- In some cases, $r$ is considered a descriptive statistic
  - $r$ is actually a direct measure of effect size:
    - It provides information about the strength of the association between two variables
    - It is a standardized measure

# Questions?

# Part 3: Hypothesis Testing with $r$

# Hypotheses

- There may be times that a correlation is the test of interest, and we can formulate associated hypotheses tests.
- There is no association between two random variables, so the null hypothesis should reflect this:
  - $H_0 : r = 0$
  - $H_{1\ two-tailed} : r \neq 0$
  - $H_{1\ one-tailed} : r > 0\ \vee\ r < 0$

# Significance Testing

Remember the key steps of hypothesis testing:

1. Compute a test statistic

2. Locate the test statistic on a distribution that reflects the probability of each test statistic value, given that H0 is true.

3. Determine whether the probability associated with your test statistic is lower than $\alpha$

# Calculation

### Compute a test statistic

- The sampling distribution for $r$ is approximately normal with a large $n$, and is $t$ distributed when $n$ is small.
    - Thus, significance is assessed using a $t$-distribution
- The $t$-statistic for a correlation is calculated as:

$$t = r\sqrt{\frac{n-2}{1-r^2}}$$

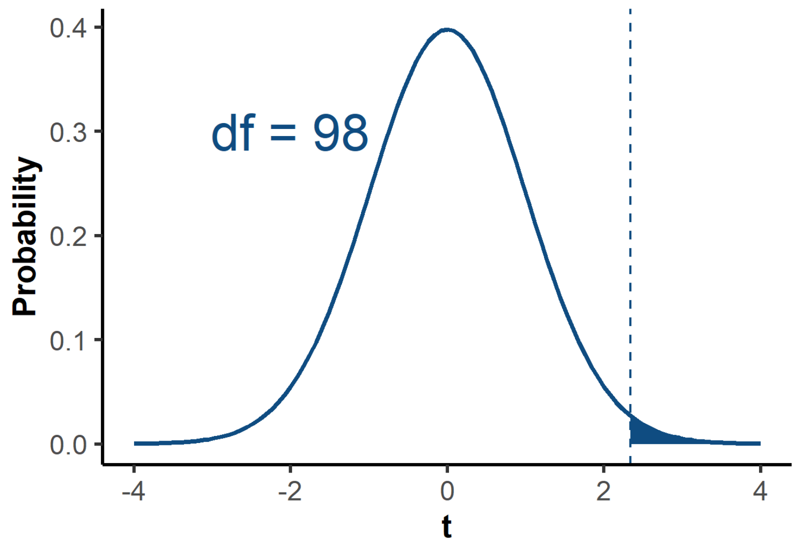
- So in our example:

$$t \;=\; 0.23\sqrt{\frac{100-2}{1-0.23^2}} \;=\; 0.23\sqrt{\frac{98}{0.95}} \;=\; 0.23\sqrt{103.16} \;=\; 2.34$$

# Is our test significant?

## Locate the test statistic on a distribution

- We have all of the pieces we need:
  - Our $t$-statistic = $2.34$
  - We use a $t$ distribution with $n-2$ degrees of freedom, so degrees of freedom = $100-2=98$
    - $n-2$: we had to calculate the means of *two* variables (e.g., in our example `daysInProgram` and `maxDistance`)
  - We will use two-tailed $\alpha$ = .05
- So now all we need is to calculate the $p$-value in order to make our decision.



# Is our test significant?

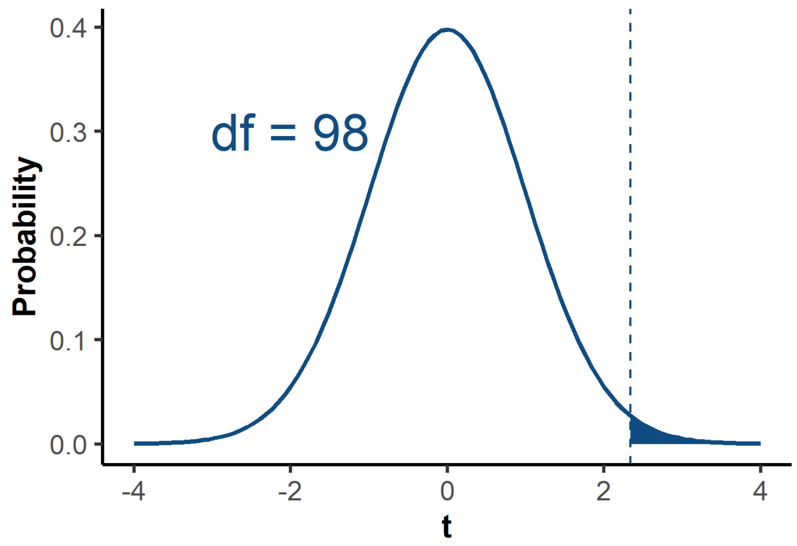
**Determine whether the probability associated with your test statistic is lower than $\alpha$**

- What proportion of the plot falls in the shaded area?

```
tibble(
  Exactp = round(2*(1-pt(2.34, 98)),2)
)
```

```
## # A tibble: 1 × 1
##   Exactp
##    <dbl>
## 1   0.02
```

- The probability that we would have a $t$-statistic at least as extreme as 2.34 if $H_0$ were true is only 0.022
  - $0.022 < .05$, so we conclude our results are significant



# Our Test: in R

- Use `cor.test()`

```
cor.test(dat$daysInProgram, dat$maxDistance,
         alternative = "two.sided")
```

```
## 
##      Pearson's product-moment correlation
## 
## data:  dat$daysInProgram and dat$maxDistance
## t = 2.3741, df = 98, p-value = 0.01954
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.03855169 0.41080493
## sample estimates:
##       cor 
## 0.2332039
```

- Note: There might be a small amount of rounding error when we compare to calculated in **R**.

# Write Up

There was a weak positive correlation between maximum distance and the number of days in program ($r = .23, t(98) = 2.37, p = .020$). These results suggested that a greater number of days in the program was positively associated with a higher maximum running distance.

# Questions?

# Part 4: Assumptions

# Assumptions of Pearson Correlation

1. Variables must be interval or ratio (continuous)
    - Knowledge of your data
    - No Likert scales!

# Assumptions of Pearson Correlation

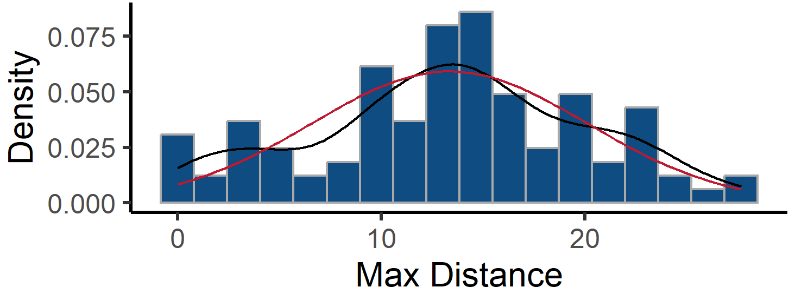
1. Variables must be interval or ratio (continuous)

2. Variables must be normally distributed

**Max Distance**

```
shapiro.test(dat$maxDistance)
```

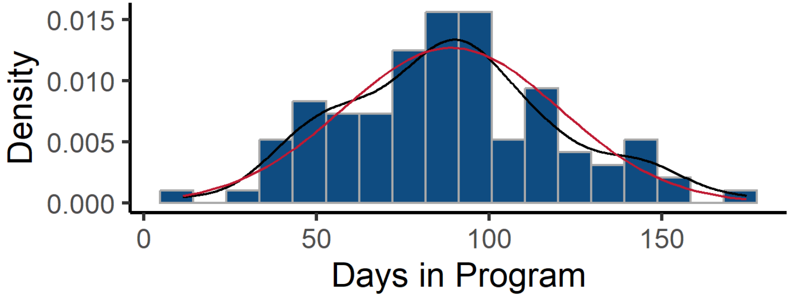
```
##
##      Shapiro-Wilk normality test
##
## data:  dat$maxDistance
## W = 0.98001, p-value = 0.1332
```




**Days in Program**

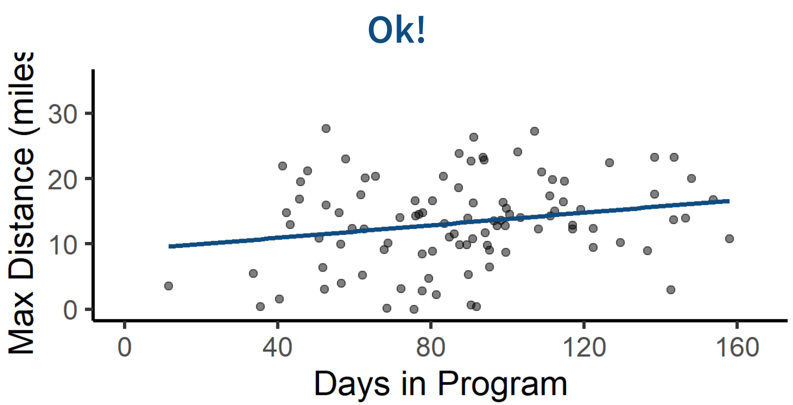
```
shapiro.test(dat$daysInProgram)
```

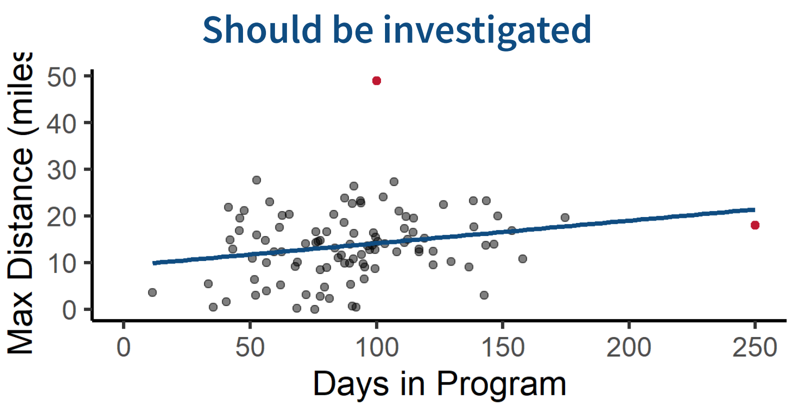
```
##
##      Shapiro-Wilk normality test
##
## data:  dat$daysInProgram
## W = 0.98833, p-value = 0.533
```



# Assumptions of Pearson Correlation

1. Variables must be interval or ratio (continuous)
2. Variables must be normally distributed
3. There must be no extreme outliers in your data

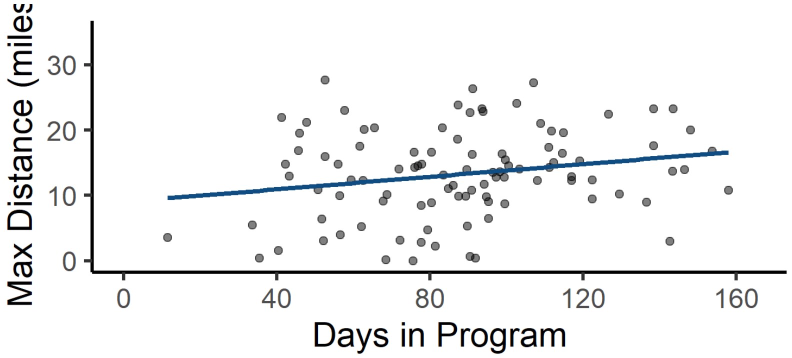


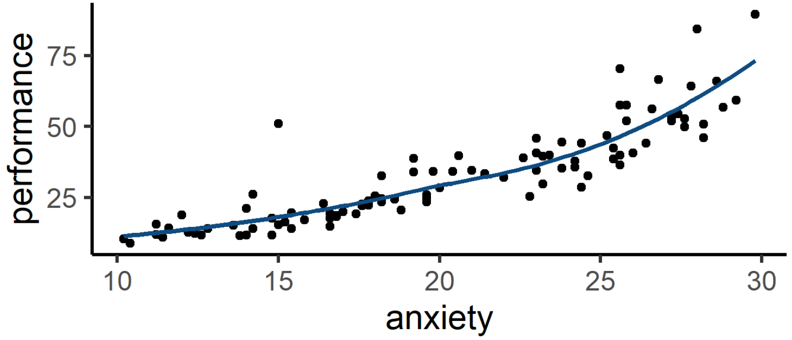
# Assumptions of Pearson Correlation

1. Variables must be interval or ratio (continuous)
2. Variables must be normally distributed.
3. There must be no extreme outliers in your data
4. The association between the two variables must be linear

**Ok!**



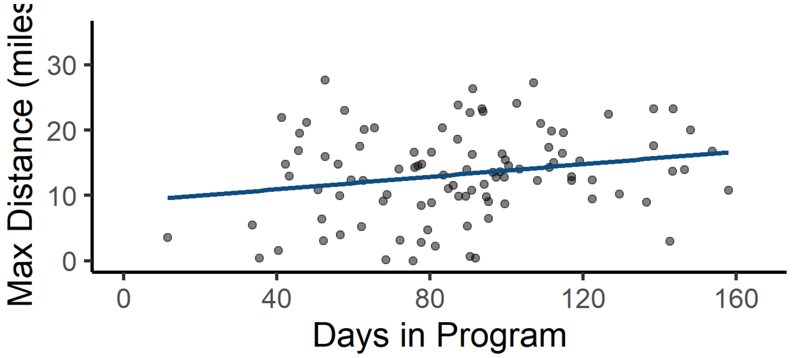

**Should be investigated**



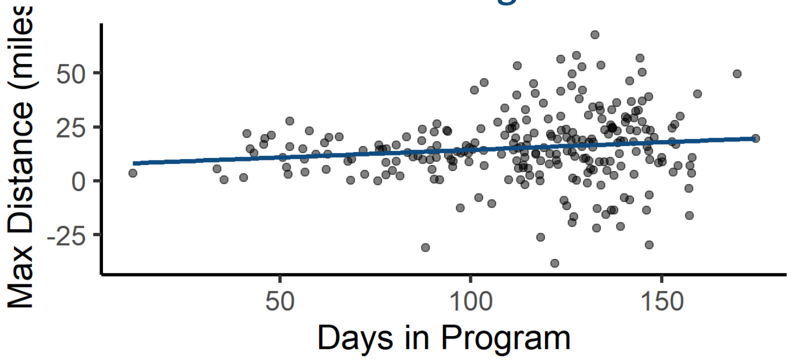
# Assumptions of Pearson Correlation

1. Variables must be interval or ratio (continuous)
2. Variables must be normally distributed.
3. There must be no extreme outliers in your data
4. The association between the two variables must be linear
5. Homoscedasticity (homogeneity of variance)

**Ok!**




**Should be investigated**



# Questions?

# Part 5: Other Types of Correlation

# Types of Correlation

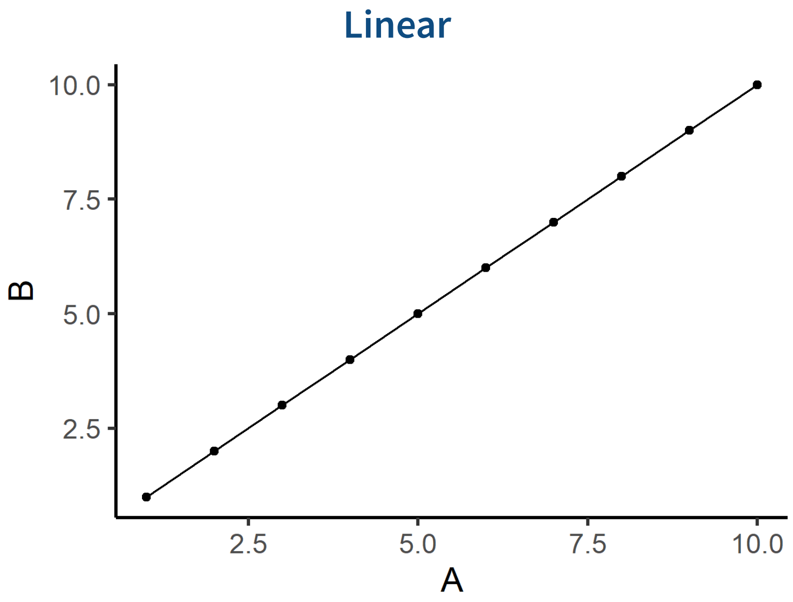
| Variable 1 | Variable 2 | Correlation Type |
|---|---|---|
| Continuous | Continuous | Pearson |
| Continuous | Categorical | Polyserial |
| Continuous | Binary | Biserial |
| Categorical | Categorical | Polychoric |
| Binary | Binary | Tetrachoric |
| Rank | Rank | Spearman |
| Nominal | Nominal | Chi-square |

# Spearman's Correlation

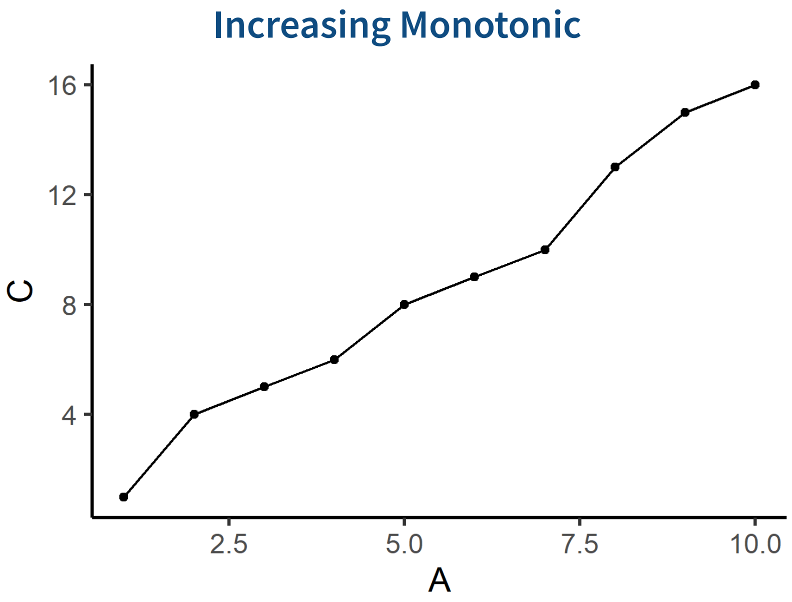
- Spearman's $\rho$ (or rank-order correlation) uses data on the rank-ordering of $x, y$ responses for each individual.
- Spearman's $\rho$ is a nonparametric version of Pearson's $r$, so it doesn't require the same constraints on your data
- When would we choose to use the Spearman correlation?
    - If our data are naturally ranked data (e.g. a survey where the task is to rank foods and drinks in terms of preference)
    - Our data are ordinal (e.g., Likert scales)
    - If the data are non-normal or skewed
    - If the data shows evidence of non-linearity

# Spearman's Correlation

- Spearman's is not testing for linear associations, it is testing for increasing monotonic association.
  - What?




A perfectly linear association between A & B

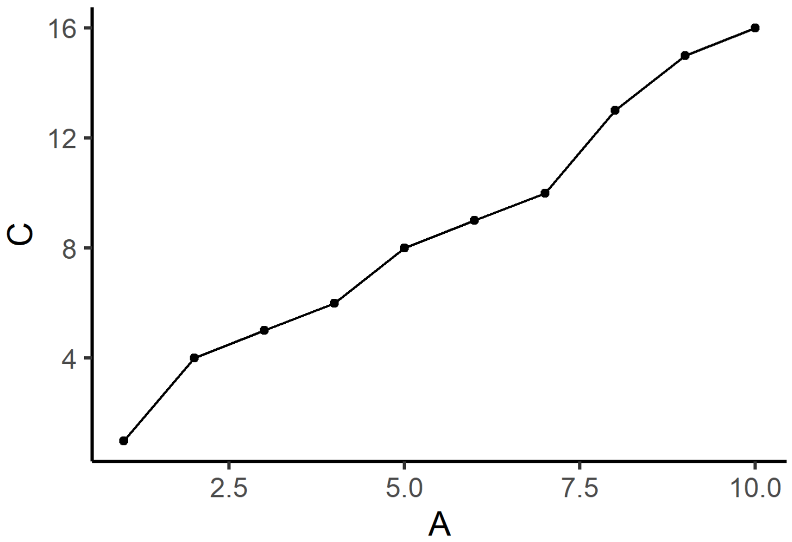



A perfectly increasing monotonic association between A & C

# Monotonic Association

- **Perfect Monotonic Association:** The rank position of all observations on Variable A is the same as the rank position of all observations on Variable C.

| ID | A | C | Rank_A | Rank_C |
|---|---|---|---|---|
| ID1 | 1 | 1 | 1 | 1 |
| ID2 | 2 | 4 | 2 | 2 |
| ID3 | 3 | 5 | 3 | 3 |
| ID4 | 4 | 6 | 4 | 4 |
| ID5 | 5 | 8 | 5 | 5 |
| ID6 | 6 | 9 | 6 | 6 |
| ID7 | 7 | 10 | 7 | 7 |
| ID8 | 8 | 13 | 8 | 8 |
| ID9 | 9 | 15 | 9 | 9 |
| ID10 | 10 | 16 | 10 | 10 |



# Calculating Spearman's $\rho$

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

- $d_i =$ rank( $x_i$ ) $-$ rank( $y_i$ )
- Steps
    1. Rank each variable from largest to smallest
    2. Calculate the difference in rank for each person on the two variables
    3. Square the difference
    4. Sum the squared values

# Calculating Spearman's $\rho$ by hand

- Imagine we want to know whether the participants' ratings (on a 1-10 scale) of the program are associated with how difficult they found the program (on a 1-10 scale):

| Names | ProgRating | Difficulty |
|---|---|---|
| Alfred | 7 | 10 |
| Bernard | 8 | 9 |
| Clarence | 4 | 8 |
| Dorothy | 2 | 5 |
| Edna | 5 | 7 |
| Flora | 3 | 1 |
| Geraldine | 1 | 4 |

| $x_i$ | $y_i$ | $x_i$ ranked | $y_i$ ranked | $di$ | $di^2$ |
|---|---|---|---|---|---|
| 7 | 10 | 6 | 7 | -1 | 1 |
| 8 | 9 | 7 | 6 | 1 | 1 |
| 4 | 8 | 4 | 5 | -1 | 1 |
| 2 | 5 | 2 | 3 | -1 | 1 |
| 5 | 7 | 5 | 4 | 1 | 1 |
| 3 | 1 | 3 | 1 | 2 | 4 |
| 1 | 4 | 1 | 2 | -1 | 1 |
| | | | | | **10** |

$$\rho = 1 - \frac{6 \cdot 10}{7(7^2 - 1)} = 1 - \frac{60}{7(49 - 1)} = 1 - \frac{60}{336} = 1 - 0.18 = 0.82$$

# Calculating Spearman's $\rho$ in R

- You can also use `cor()` and `cor.test()` to calculate Spearman's $\rho$ in R
  - Just need to update the `method =` argument
- Necessary to use when you have ties in your ranks (i.e., your ranks are not unique)

```
cor.test(ratings$ProgRating, ratings$Difficulty,
         method="spearman",
         alternative = "two.sided")
```

```
##
##      Spearman's rank correlation rho
##
## data:  ratings$ProgRating and ratings$Difficulty
## S = 10, p-value = 0.03413
## alternative hypothesis: true rho is not equal to 0
## sample estimates:
##       rho
## 0.8214286
```

# Summary

- Today we have covered:
  - The differences between variance, covariance, and correlation
  - How to calculate both covariance and correlation
  - How to interpret both the correlation coefficient and the results of the associated significance test
  - Other methods for correlation and calculated Spearman's $\rho$

# Tasks & Next Week

### Tasks

- Go to your lab and work on the assessed report
- Submit the assessed report by 12 noon on Friday 29th March 2024, see report instructions
- Complete the Peer Assessment of Contribution form
  - The form closes at 12 noon on the 5th of April 2024
  - Find it on the course LEARN page > Assessment > DAPR1 Peer Assessment
- Go to office hours if you have questions
  - Emma's Office Hours = Tuesdays 10:30-11:30 in G15, 7 George Square
- Complete any assigned reading

### Next week (week beginning 1st April)

- There will be no lectures
- The labs will still go ahead
  - In the labs you will work through mock exam questions
  - Don't forget to bring a calculator, a pencil, and an eraser with you
- Next week will be the last week of labs (and of the course)